OPTIMAL BUS DISPATCHING POLICY UNDER VARIABLE DEMAND OVER TIME AND ROUTE LENGTH

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ABSTRACT

The problems of scheduling and schedule co-ordination in bus operations have conflicting objectives related to user's cost and operator's cost. Passengers would like to have public bus services where there is less waiting time. Operators on the other hand would like to earn profit with lesser vehicle operating cost and a minimum number of buses.

In developing countries where overloading of buses has long been considered necessary to ensure bus travel remains affordable to most socioeconomic groups, bus operators would in addition to larger headways, like to have higher load factors to increase revenue even though passengers would prefer less load factors as it provides a more comfortable journey. All these factors are further constrained by the fare levels, which may not make the revenue adequate to operate at the most economically optimal frequency and load factor.

This paper considers a method that is an extension to Newell's Optimal Dispatching Policy, to determine a fleet size and dispatching rate based on both operator's cost and user's cost including the disutility of standing, in order to arrive at a global cost optimum. It further investigates the financial viability of providing such a service and sets out a financial viability domain within which optimization can occur in practice. If the resulting dispatching rate is lower and does not fall within the domain of financial viability, then operating subsidies are considered necessary to maintain the economically optimum dispatching rate.

This method to compute optimized dispatching rates is based on screen-line counts across given locations along a bus routes used in conjunction with a limited sample of on-board boarding and alighting surveys. Passenger revenues have been computed by a process of multiplication of the rationalized origin-destination matrix by the fare for distance travelled between the respective origins and destinations. Indicators have also been developed to determine average trip lengths for each route and average revenue per passenger together with the points of maximum capacity along the route. These indicators describe the nature of the demand that the bus route serves.

The screen line counts provide the hourly variation in demand over a bus route throughout the day, which has been expressed in terms of a polynomial equation to determine the variation of demand over different time periods. By combining both functions, a composite function has been developed to determine; the daily passenger demand on a given route; the total revenue for operators, the average load factor and locations on the route where maximum loading occurs.

INTRODUCTION

Dispatching rate on a bus route is pertinent to both its primary stakeholders, the operators and commuters alike. For the sustenance of the system, the priority and most obvious requirement is that the operator has to maintain financial viability of this operation. If the dispatch rate (Ǿ) defined in terms of buses dispatched over a route per hour is high, his operating cost increases and consequently financial viability is affected significantly unless there are adequate passengers yielding revenue, which covers the cost of dispatching at rate Ǿ. In this situation his financial viability depends on two other factors; (a) the load factor of buses and (b) revenue or fare level. It is therefore the objective function for operators to determine a dispatching rate Ǿ within the parameters of the route namely, the passenger demand, allowed loading factor and fare level. The operators' objective is therefore to dispatch buses at a rate lower than the financially feasible dispatching rate Ǿf. Therefore, we have a constraint:

$$Ǿ \leq Ǿ_f$$

In a regulatory regime, where bus fares are fully deregulated, the operator has the option of increasing his fare level so that, a revenue optimizing commercial decision is arrived at by trading off lower dispatching rates with higher fares. However, in an operating regime where fares are fully regulated this freedom is not with the operator. Therefore, financial viability is usually achieved by the operator determining a dispatching rate determined on the maximum load factor allowed by law or in the absence of such, by the physical capacity of the bus. Under such conditions, over loading increases, dispatching rates reduce and consequently the quality of service for the other stakeholder-the passenger deteriorates.

This brings us to the factors that affect the cost of the passengers usually recognized in terms of fare, waiting time, travel time and load factor. As any one or more of these parameters increases, so does the cost to the passenger. Thus higher waiting times when dispatching rate increases will diminish the utility (benefit) he obtained from the service, thereby increasing his generalized cost, which in turn could reduce the passengers on the route.

If one were to relate to a hypothetical condition where the absolute expectation of the commuter is satisfied, where a bus is available whenever he wishes to travel, then waiting time becomes zero. This ideal situation is then comparable to individual private transport. It is evident then that these two primary stakeholders in bus services have conflicting objectives towards dispatching times. Newell, 1971 introduced the concept of social cost by taking the optimum of the aggregated costs for both stakeholders. This approach however assumes that the:

- Operator is able to earn adequate revenue to cover cost of operating the service at the optimally determined dispatching rate.
- Operator is able to either vary his fare with variations in costs, passenger demand or qualitative factors such as lower waiting time or load levels as may be

demanded by the passengers or that he would be subsidized by the government to compensate any loss in revenue.

- Demand would not vary along the length of the route during the traffic day

Recent work on (i) and (ii) by Piyadasa and Kumarage (2002) has concluded that the financially optimum dispatching rate has to be always lower than the economically optimum dispatching rate and points out that two strategies could be employed to reach an universal optimum where both financial and economic optimum dispatching rates are equal at a unique point. This paper therefore investigates more specifically item (iii) of the above matters but will use the theory developed for determining such a global optimum.

Financial Constraints on Dispatching Buses on Optimized Economic Criteria

Piyadasa and Kumarage, (2002) developed the argument that when taking the revenue of an entire route, the overall revenue does not change with respect to changes in dispatching rate and correspondingly the headways between buses dispatched. This is just because supply increases demand does not increase and as such it is reasonable to assume that revenue or financial return to the bus operator remains a constant FRO. This is shown in Figure 1 along with the variation of the financial costs of operation (FCO) to the operator, which considers that as dispatching rate increases, the number of buses required and the number of trips made will increase thereby increasing the cost to the operator as well as to society in terms of resource use. The curve in Figure 1 follows a diminishing cost curve (Newell, 1971) and there exists a point hf in headways between buses, after which increasing the dispatching rate becomes unfeasible.

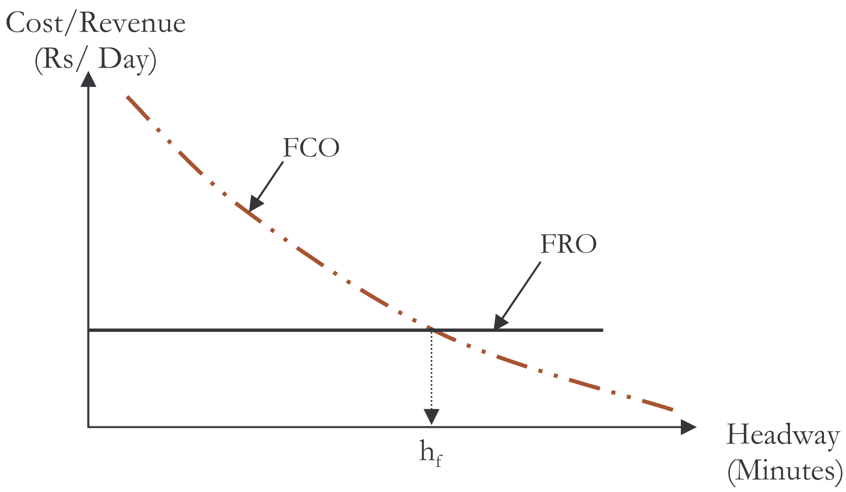

Figure 1: Optimum Headway of Bus Dispatching Based on Financial Revenue and Cost to the Operator

Let us now develop the economic cost curve that includes costs to passengers.

This is shown in Figure 2, where

ECO - Economic Cost of Operation

FRO - Financial Revenue to Operator

ECP	-	Economic Cost to Passengers
h_o	-	Optimum headway
$h_f^{'}$	-	Financially feasible headway to the right of h_o ($> h_f^{'}$)
$h_f^{''}$	-	Financially feasible headway to the left of h_o ($< h_f^{''}$)

Following the same rationale, if one were assumed that the relationship between financial costs and economic costs are linear, (i.e. ECO ∞ FCO), then we have an economically optimum headway, hO, which is the headway corresponding to the desired dispatching rate for society.

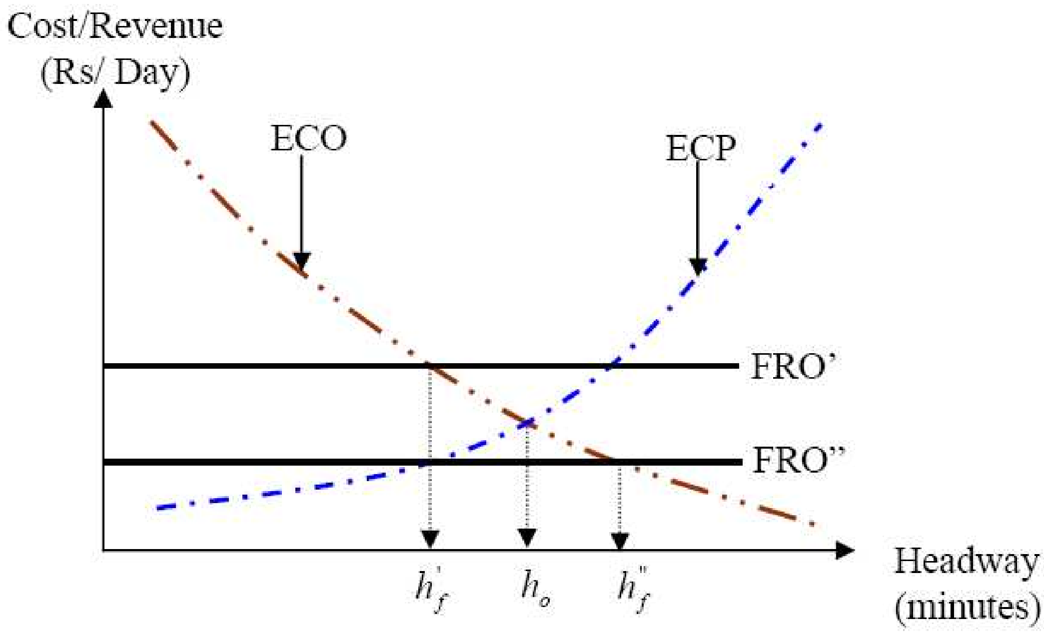

Figure 2: Constraints of Financial Revenue on Optimum Dispatching Rate

It is reasonable to assume that any operator would prefer to have a dispatching rate that would lead to an operating headway to the right of h_o (e.g. $h_f^{''}$) which will effectively lower his costs so that with fixed revenues, profits will then increase. Therefore, an operator's preferred dispatching rate is represented by corresponding headways to the right of ho. In a situation where cross-subsidy is available between different times of the day, the average ho may differ when buses are being dispatched at headways $h_f^{'}$ which is less than what is optimal (i.e. to the left of ho) during times of heavy demand and at headways higher than ho during lean periods of demand such as during off-peak periods, early morning or late nights. This forms an internal cross subsidy within a route managed by the operator within his overall financial viability. In developing countries, regulators may want to specify such periods of minimum headway as a policy to fulfil certain minimum affordability conditions as a social requirement.

The estimate of the total revenue on a bus route is easily obtained when there is only one operator on the route. However, in bus transport systems where ownership is in the hands of individual owner-operators, information on revenue may not be readily available. It therefore, becomes necessary to have estimate-based criteria, which could be adopted to determine the average daily passenger demand on a route as well as its variations against time and space.

According to Strathman et al (1999), changes in headway variation and run times were used to estimate the initial benefits of this kind of system with respect to operation costs, passenger waiting and passenger travel time. Following Hounsell and McLeod (1998), headway variation was also used to derive a measure of excess waiting time that passengers had to experience due to unreliable service.

Demand Estimation on Route Served by Multiple Operators

Abkowitz and Engelstein (1984) have developed a linear regression model to estimate the mean running time of a bus route. In this model, passenger boarding and passenger alighting behaviour are considered as independent variables. This model has shown that the mean running time is highly influenced by factors such as boarding and alighting, trip distance, time of the day and direction of travel.

Abkowitz and Tozzi (1986) have also developed another mathematical model to investigate the impact of five boarding and alighting profiles on the effectiveness of headway based control. These profiles specified that;

- Passengers board at the beginning and alight at the end of the route (one to one)
- Passengers board at the beginning and alight in the middle and at the end of the route (one to specified stops).
- Passengers board at the beginning and alight in the middle of the route (one to one).
- Passengers board and alight uniformly along the route (specified to specified).
- Passengers board in the middle and alight at the end of the route (one to one).

All the above refer to situations where the locations for boarding and alighting along a route are specified. However, in local bus services, such stops cannot be specified and operations are mostly many to many i.e; passengers may board at any point and alight at any other point along the route. This form of most generalized boarding and alighting pattern is used to estimate the demand function of a route over its length.

$$Y = f(t) \tag{1}$$

The demand for a route along its entire length therefore has to be studied as a demand function represented in terms of a parabolic curve is the kth degree. Thus, the demand for passengers on a particular route with respect to the time of the day (represented as x in the equation 2) would be a polynomial curve where the generalization form of the equation of the demand curve would be represented as;

$$y = a_0 + a_1 x + a_2 x^2 + \dots\dots\dots + a_k x^k \tag{2}$$

The Principle of Least Squares method is used to find out the constants a_1, a_2, a_3, …, and a_k. To approximate the given set of data, (x_1,y_1), (x_2,y_2),…, (x_n,y_n), where $n \geq k+1$, the best fitting curve of predicted demand, $y= f(t)$, has the least square error, (i.e. the residual of equation (2)) is given by

$$R^2 = \sum_{i=1}^{n}\left[y_i - (a_0 + a_1 x_i + a_2 x_i^{\ 2} + ... + a_k x_i^{\ k})\right]^2 \tag{3}$$

It is noted that a_1, a_2, a_3, …, and a_k are unknown coefficients while x_i and y_i are given as time value and demand data respectively. To obtain the least square error, the unknown coefficients a_1, a_2, a_3, …, and a_k must yield zero in first partial derivatives.

$$\frac{\partial(R^2)}{\partial a_0} = -2\sum_{i=1}^{n}\left[y_i - (a_0 + a_1 x_i + a_2 x_i^{\ 2} + ... + a_k x_i^{\ k})\right] = 0 \tag{4}$$

$$\frac{\partial(R^2)}{\partial a_1} = -2\sum_{i=1}^{n}\left[y_i - (a_0 + a_1 x_i + a_2 x_i^{\ 2} + ... + a_k x_i^{\ k})\right]x = 0 \tag{5}$$

…

$$\frac{\partial(R^2)}{\partial a_k} = -2\sum_{i=1}^{n}\left[y_i - (a_0 + a_1 x_i + a_2 x_i^{\ 2} + ... + a_k x_i^{\ k})\right]x^k = 0 \tag{6}$$

These lead to the equations

$$\sum_{i=1}^{n} y_i = na_0 + a_1 \sum_{i=1}^{n} x_i + ... + a_k \sum_{i=1}^{n} x_i^k \tag{7}$$

$$\sum_{i=1}^{n} x_i y_i = a_0 \sum_{i=1}^{n} x_i + a_1 \sum_{i=1}^{n} x_i^2 + ... + a_k \sum_{i=1}^{n} x_i^{k+1} \tag{8}$$

. . . . .

$$\sum_{i=1}^{n} x_i^k y_i = a_0 \sum_{i=1}^{n} x_i^k + a_1 \sum_{i=1}^{n} x_i^{k+1} + ... + a_k \sum_{i=1}^{n} x_i^{2k} \tag{9}$$

Equation 7, 8 and 9 can also be represented in matrix form, as shown in equation (10)

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_i & \cdots & \sum_{i=1}^{n} x_i^k \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & \cdots & \sum_{i=1}^{n} x_i^{k+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{n} x_i^k & \sum_{i=1}^{n} x_i^{k+1} & \cdots & \sum_{i=1}^{n} x_i^{2k} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ q_k \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \\ \vdots \\ \sum_{i=1}^{n} x_i^k y_i \end{bmatrix} \tag{10}$$

This matrix equation can be solved numerically and the answer gives the values for the unknown constants a_1, a_2, a_3,…, and a_k. Consequently the best fitted curve of the predicted demand curve, $y=f(t)$, is available for further analysis.

The equation of the predicted demand curve illustrated as shown in Figure 3, which gives the number of passengers carried on all buses across any given point along the route, over time period t.

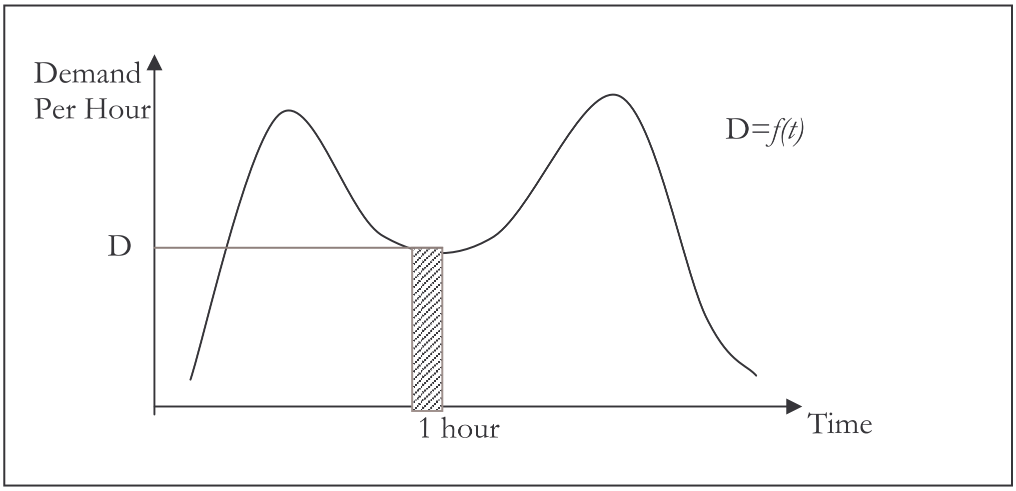

Figure 3: Demand of Passengers on a Route over Traffic Day

Computation of Revenue on a Route Served by Many operators

In general, there are three different events involved in the carriage of a passenger that is implicitly built into the fare computation on most routes. These are:

- A passenger boards the bus
- A passenger alights from the bus
- A passenger is carried past a stop or section of the route.

If there are n numbers of stops on a particular route, then there would be n-1 number of sections, where a section would be defined as the section of route between two adjacent stops. Figure 4 illustrates this along with Ai, Bi (where i = 1, 2, … , n) representing the respective number of people actually boarding and alighting at that stop.

If the fare computation is given in terms of passenger charge for boarding (β), alighting (α) and for carrying past one bus stop (γ), the revenue R of a single bus at a any particular location on the route can be represented as:

$$R = \sum_{i=1}^{n}(A_i \times \alpha) + \sum_{i=1}^{n}(B_i \times \beta) + \sum_{i=1}^{n-1}(X_i - A_{i+1}) \times \gamma \quad (11)$$

Where Ai = number of passengers alighting in section i,

B_i = number of passengers boarding in section i and

X_i = number of passengers carried within section i

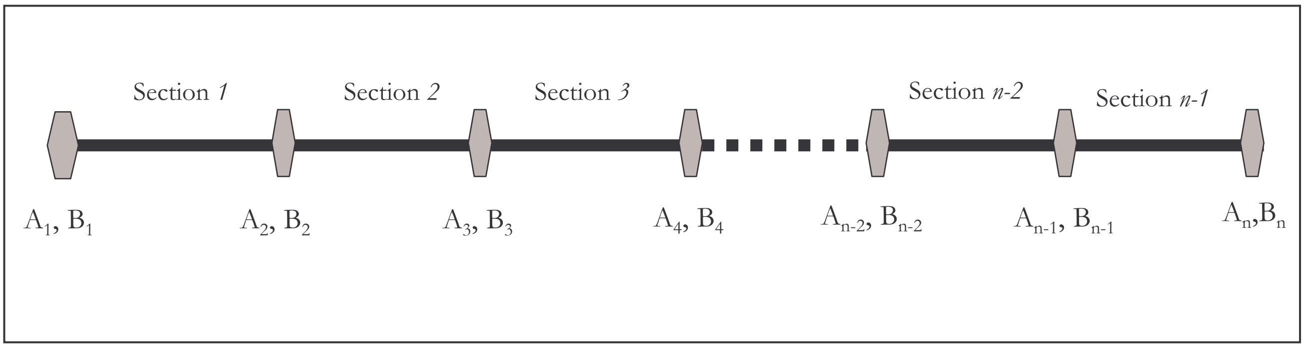

Figure 4: Boarding & Alighting Patterns on a Bus Route with n Stops

In the absence of data from on board counts, estimates can be made at locations along the route, where the revenue of a bus route can be determined from a sample of buses for which boarding and alighting surveys have been carried at that location. This we will represent in terms of a variable 'Revenue Ratio' defined at the ratio of revenue between that which is estimated for a particular trip, i, (Ri), and the maximum possible revenue earnable in jth location assuming that all passengers at that location were to travel the entire distance (Rj,max). The Revenue Ratio can thus be written as:

Revenue Ratio (i,j) , $k_{t(i,j)} = \dfrac{\mathrm{R}_{\mathrm{i}}}{\mathrm{R}_{\mathrm{j,max}}}$ (12)

Where;

$k_{t(i,j)}$ is the Revenue Ratio for i^{th} trip at j^{th} location.

Average of all $k_{t(i,j)}$ over a route and traffic day, is called "Average Revenue Ratio, k_t " as given in equation 13.

$$k_t = \frac{\sum_{\mathrm{i=1}}^{\mathrm{l}} \sum_{\mathrm{j=1}}^{\mathrm{m}} k_{t(i,j)}}{(\mathrm{i, j})} \qquad (13)$$

Where l is number of trips and m is number of locations

If we assume that the demand is uniform over the length of the route, then the financial revenue from bus operations on the route during the trip (R_t), is given by the expression:

$$R_t = k_t f_t D_t \qquad (14)$$

Where f_t = fare to travel total length of the route and

D_t = total demand of passenger per trip.

k_t = Average Revenue Ratio of *(i,j)*

But since we have already assumed that demand for passengers over time, is a function of time (t), then:

$$D = f(t) \tag{15}$$

Therefore, the revenue of any trip given in equation 14, can be calculated using the Average Revenue Ratio given in equations 13 by multiplying the demand function at particular location represented by equation 15 so that total fare of the route is

$$R_t = f(t)_t k_t f_t \tag{16}$$

Therefore, the Financial Revenue to all operators on the route per day is given by:

$$\sum R_t = k_t f_t \sum D_t \tag{17}$$

Validation

The 24 km long urban bus route, Panadura – Nugegoda (Route # 183), is used as a case study. The analysis uses loading data at a mid way location "Ratmalana" on the route. Table 1 shows the summary of loading data at this location in one-hour interval throughout the traffic day.

Table 1 shows variation in the demand pattern at a point on a route in during the entire traffic day from 6AM to 6PM. In order to fit the demand data into a parabolic curve, the order of the polynomial function is assumed to be 6 and n =13 which denotes the number of data samples. The unknown constants of the polynomial equation can be found when the respective data from Table 1 is substituted in the equation 10, as time value xi and demand value yi, so that:

$$D = 0.0016t^6 - 0.093t^5 + 1.71t^4 - 12.26t^3 + 24.29t^2 + 32.56t + 376.8 \tag{18}$$

Table 1: Loading Survey Data

Time (starting)	No. of Buses	Demand (per hour)	Supply (seats per hour)	Load Factor
6:00	3	131	121	1.1
7:00	9	440	396	1.1
8:00	11	540	495	1.1
9:00	7	410	293	1.4
10:00	6	320	258	1.2
11:00	5	248	225	1.1
12:00	6	216	270	0.8
13:00	7	350	295	1.2
14:00	6	212	252	0.8
15:00	6	370	270	1.4
16:00	6	310	270	1.1
17:00	6	350	261	1.3
18:00	3	109	116	0.9

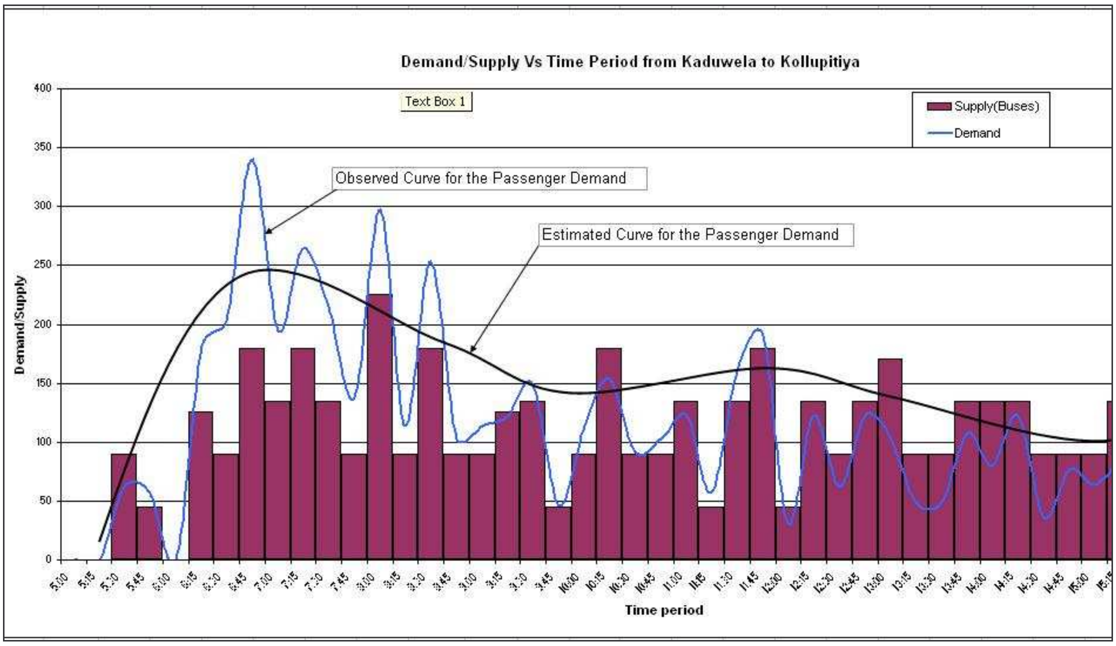

Figure 5: Demand of Passengers on a Route over period *t*

Figure 5 superimposes the normalized demand curve on the observed hourly demand and the existing hourly supply in terms of seats dispatched per hour, which in turn gives the dispatching rate.

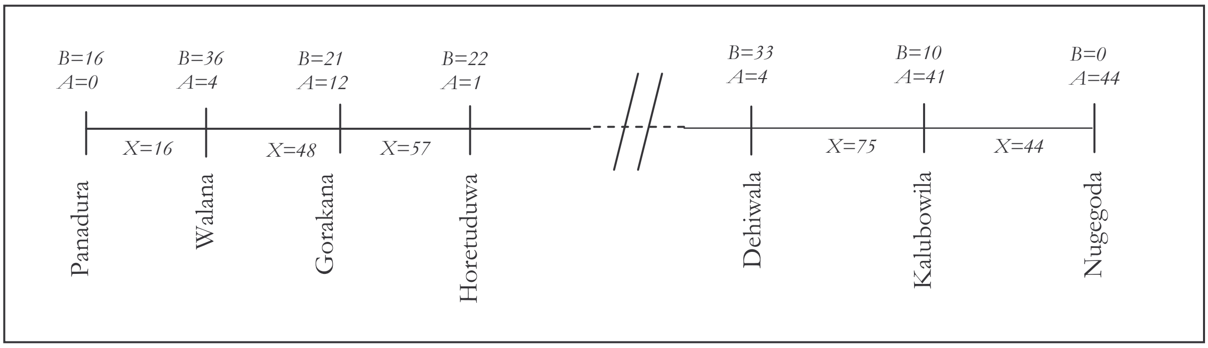

Figure 6: Summary of Passenger Boarding & Lighting for a Single Trip

The boarding and alighting survey data of a single bus trip on the same route is shown below, where the fare section is approximately 2 kms in length is given in Table 2

Figure 6 gives the corresponding number of passengers in each section computed from the following table.

Table 2: Passenger Boarding & Alighting Data for a Single Trip

Stop Details		**Total**		**Carried to**
Section	**Time**	**Boarding**	**Alighting**	**next section**
1. Panadura	6.51	16	0	16
2. Walana	7.00	36	4	48
3. Gorakana	7.08	21	12	57
4. Horetuduwa	7.12	22	1	78
5. Moratuwa	7.16	15	26	67
6. Katubedda	7.27	21	25	63
7. Ratmalana	7.36	7	18	52
8. Mt.Lavinia	7.45	15	21	46
9. Dehiwala	7.57	33	4	75
10. Kalubowila	8.10	10	41	44
11. Nugegoda	8.20	0	44	
TOTAL		**196**	**196**	**546**

In this particular route total of ten section samples are taken (i.e: (n-1) = 10) and therefore by applying in equation (11).

$$R = 196 \times \alpha + 196 \times \beta + 546 \times \gamma \tag{19}$$

The expression above reveals that (a) the total number of passengers who boarded the bus was 196 and (b) the aggregated number of sections they travelled across was 546. With minimum fare being Rs 5.00 (around US 5 cents) the value for the constants α, β are each assumed to be ½ of this value. The value for γ is calculated taking the average fare per section for this route and found to be approximately Rs. 1.80 per section. Therefore, the total revenue for this bus trip is Rs 1,610 computed as follows:

R = 196x2.50 + 196 x 2.50+546x1.80

R = 1,962.80 (Rupees)

Summing the demand over the traffic day and multiplying with the corresponding revenue per trip provides the total financial revenue to all operators on the route.

Revenue Ratios are calculated for different trips during the different times of a traffic day for each of the locations where roadside loading surveys have been carried out. The calculation of Revenue Ratio for the ith trip for the three loading survey locations on this particular route is as follows;

$$k_{t,i} = \left(\frac{\mathrm{R}_{(B.\&A,i)}}{\mathrm{R}_{\text{LS-1}}} + \frac{\mathrm{R}_{(B.\&A,i)}}{\mathrm{R}_{\text{LS-2}}} + \frac{\mathrm{R}_{(B.\&A,i)}}{\mathrm{R}_{\text{LS-3}}} \right) / 3 \qquad (20)$$

$$k_{t,i} = \left(\frac{1{,}962.80}{48 \times 19.50} + \frac{1{,}962.80}{52 \times 19.50} + \frac{1{,}962.8}{44 \times 19.50} \right) / 3$$

$$k_{t,i} = 2.11$$

The calculated revenue ratio for the i^{th} trip for the bus route # 183, Panadura – Nugegoda is 2.11. The total average revenue ratio for the route is average of all *i* numbers of trips as indicating in equation 21;

$$k_t = \left(\frac{k_{t,1} + k_{t,2} + \ldots + k_{t,i}}{i} \right) \qquad (21)$$

The revenue of any trip occurring in time *t* on this route can be found from the equation 22.

$$R_t = k_t f_t [0.0016t^6 - 0.093t^5 + 1.71t^4 - 12.26t^3 + 24.29t^2 + 32.56t + 376.8] \quad \text{-(22)}$$

Summing the revenue of all trips occurring during the traffic day would give the total revenue on the route. Therefore, the total route revenue to operators FRO can be computed using equation 17 and is given as:

$$FRO = k_t f_t \int_{t1}^{t2} [0.0016t^6 - 0.093t^5 + 1.71t^4 - 12.26t^3 + 24.29t^2 + 32.56t + 376.8] \qquad (23)$$

Where

t_1 and t_2 are the starting time and finishing time of the traffic day.

Dispatching to ensure Operator's Financial Viability

As discussed earlier, although the demand varies from trip to trip it would not change the total revenue to all operators on the route. Figure 7 shows the curve FRO with corresponding ECO and ECP curves and the total economic cost given by curve TEC. For the route in question, the observed average daily dispatching rate is 81 buses over the traffic day of 12 hours giving an average headway of 10 minutes. The observed variation as shown in Table 1 is between a low of 3 buses to a high of 11 buses per hour during the peak period.

The economic cost curves for bus operations (ECO) and passenger costs (ECP) and the total economic cost to society (TEC) show that the optimum headway for this route should be 7 minutes. This means that the actual average headway ha on the route is to the right of ho. This means that either fare levels are inadequate or the operators are making super normal profits at the expense of increased economic cost to passengers. On examination of the revenue for this route as discussed above, the FRO curve is seen to intercept the ECO curve at hf which is the headway at which the route operations become financially feasible. Thus we have a condition that ha > hf > ho. This means two things namely; (a) that fare levels are insufficient and (b) that operators are still making excess profits.

It is therefore possible to reschedule timetables to reduce headways from ha to hf without an increase in fare. However in order to ensure that the route operates at an economic optimum headway of ho, fare levels have to be increased by a percentage of (FRO' – FRO)/FRO or alternately the operators should be given an operating subsidy to that value of Δf which is FRO'- FRO .

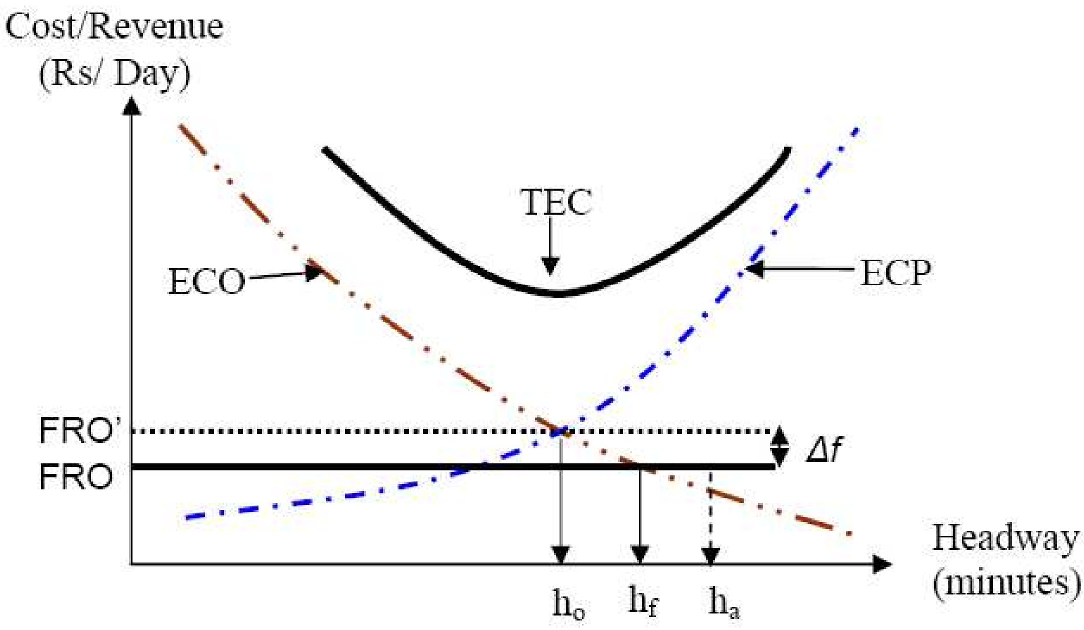

Figure 7: Effect to the Headway with Revenue Increment

CONCLUSIONS

The expectation of passengers and operators exhibits a fundamental conflict in transport planning. In order to solve this conflict the transport planner needs to arrive at an acceptable compromise by equally treating the two incompatible planning objectives.

The method presented in this paper is an extension to Newell's optimization method of determining optimal headways by computing revenue variations along a route in both time and space in order to determine the financial viability of such an optimal headway computed as a social cost to the economy. The paper provides a method for computing fares in routes where there are multiple operators and as such, this method may be used for any type of bus route.

A case study is used to compute these values and the paper shows how the existing headways is higher than the financially viable headway and much higher than the economically optimum headway. The paper concludes by showing the extend to which operational improvements could be effected in order to reduce waiting times for passengers and what revenue based changes are required in order to ensure that the actual operating headway is equal to the economically optimum headway.

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